

CSC 311—Theory of Computation

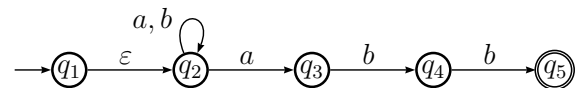
Tutorial on DFAs, NFAs, regular expressions, regular grammar, closure of regular languages, context-free grammars, non-deterministic push-down automata, Turing machines, etc.

Tutorial 2

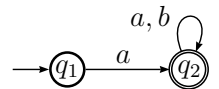
Second Semester 2017

Assume that $\Sigma = \{a, b\}$ when it is not otherwise given.

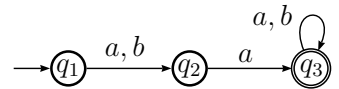
- Given the regular expression $(a|b)^*abb$, the corresponding NFA is given on the right. Rewrite this NFA into its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



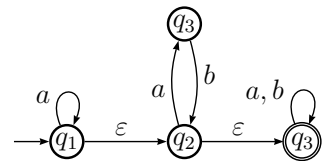
- Given the NFA N with $L(N) = \{w \in \Sigma^* | w \text{ starts with } a\}$, as the regular expression $a(a+b)^*$, first confirm that its NFA is as shown on the right then derive its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



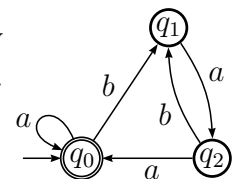
- $L(N) = \{w \in \Sigma^* | w \text{ has } a \text{ as its 2nd letter}\}$ is the language of the NFA N . Write its corresponding regular expression. First confirm that its NFA is as shown on the right then derive its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



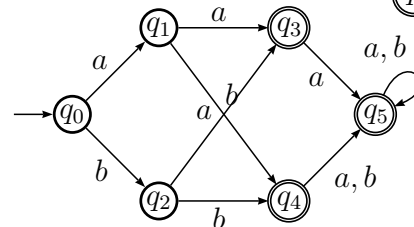
- Consider NFA N diagrammed on the right. Describe $L(N) = \{w \in \Sigma^* | w \dots\}$. Derive its corresponding DFA and write the resulting DFA both as a transition table and as a transition diagram.



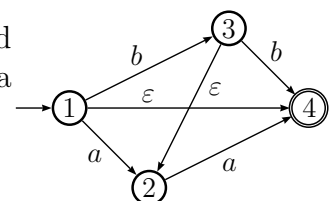
- Give a regular expression for this NFA given as a transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



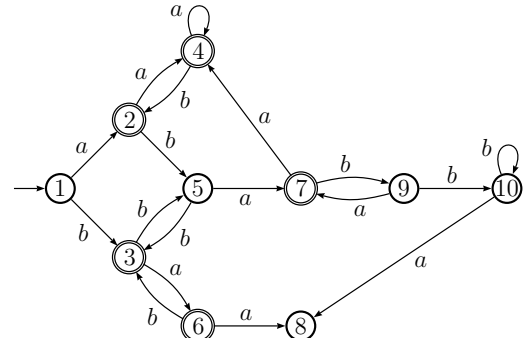
- Minimize the DFA given by this transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



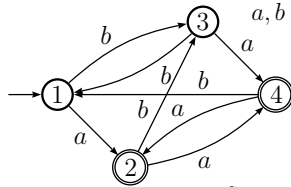
- Rewrite the NFA given by the transition graph as a DFA, and then minimize the DFA. Show the resulting DFA both as a transition table and as a transition diagram.



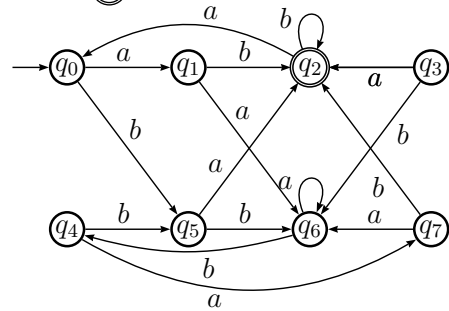
8. Minimize the DFA given by the transition graph (García et al., 2011). Show the resulting DFA both as a transition table and as a transition diagram.



9. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



10. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 155).



11. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 164).

	a	b
→ 0	1	0
1	0	3
2	3	1
*3	3	0
4	3	5
5	6	4
6	5	6
7	6	3

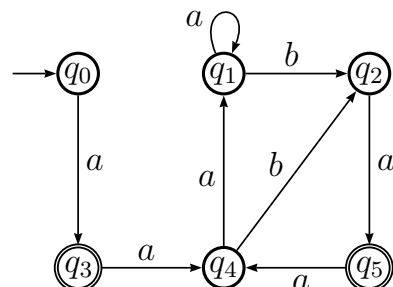
	a	b
→ 0	1	4
1	2	5
2	3	7
*3	4	7
4	5	8
5	6	1
6	7	1
7	8	2
8	0	4

12. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 165).

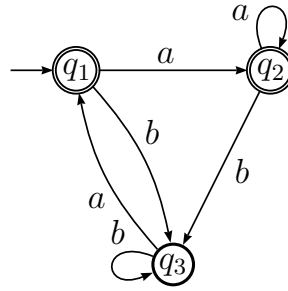
	a	b
→ 1	2	5
2	3	6
*3	4	8
4	5	8
5	6	9
6	7	2
7	8	2
8	9	3
*9	1	5

13. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 165).

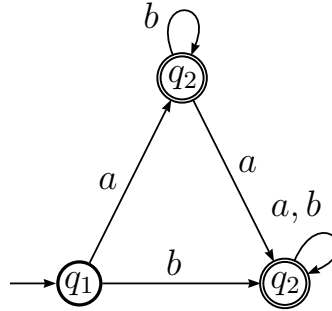
14. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Valmari, 2010). Note that all the missing transitions lead to a “dead” or “sink” state, which is not shown in the state diagram.



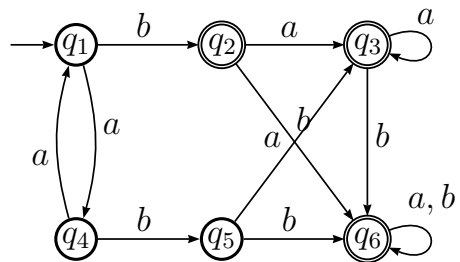
15. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 158).



16. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



17. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



18. Show that the sentence `Jim trains art students` has a derivation in the following grammar. Give a parse tree for the sentence.

$$\begin{aligned}
 S &\rightarrow NV_P \\
 N &\rightarrow NN \\
 V_P &\rightarrow VN \\
 N &\rightarrow \text{students} \mid \text{Jim} \mid \text{art} \mid \text{trains} \\
 V &\rightarrow \text{trains}
 \end{aligned}$$

19. Let G be the following grammar

$$\begin{aligned}
 S &\rightarrow AB \mid BC \\
 A &\rightarrow BA \mid a \\
 B &\rightarrow CC \mid b \\
 C &\rightarrow AB \mid a
 \end{aligned}$$

Use the CYK algorithm to determine whether $w = \text{baaba}$ is in $L(G)$. Give a parse tree for w , and its rightmost derivation from S

20. Given the grammar G with start symbol E in CNF use the CYK algorithm to determine whether $w = (x + x) * x$ is in G . E is the start variable.

$$\begin{aligned} E &\rightarrow EF \mid LR \mid x \\ F &\rightarrow ME \mid CE \\ G &\rightarrow EQ \\ P &\rightarrow + \\ T &\rightarrow * \\ L &\rightarrow (\\ R &\rightarrow) \end{aligned}$$

21. Given the grammar G with start symbol S in CNF use the CYK algorithm to determine whether $w = ababa$ is in $L(G)$.

$$\begin{aligned} S &\rightarrow AT \mid AU \mid \varepsilon \\ T &\rightarrow UB \mid b \\ U &\rightarrow AT \mid UT \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

22. Show the CYK algorithm for the CNF grammar G with start symbol S by showing that $w = ababa$ is or is not in $L(G)$.

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

23. What is the language \mathcal{L} that is derived from S where

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aA \mid a \\ B &\rightarrow AB? \end{aligned}$$

24. Write python code to read in the specifications for an NFA or DFA and then turns it into a DFA if it is not already one. Your specifications should give the NFA/DFA in the form of a *state table* with numerically numbered states and single characters as the alphabet. Use a python's sets, dictionary and regular expression package `re`.

25. Write python code which which *minimizes* your DFA.

26. Write python code to *turn a context-free grammar into Chomsky-normal form* if it is not yet in that form.
27. Write python code to *implement the CYK algorithm*, given a grammar in Chomsky normal form. It should read a grammar and run CYK to determine if some strings are accepted or rejected.
28. Develop code to *implement a PDA* from specifications, and use it to show that the PDA accepts the palindromes language.
29. Develop code to *implement a Turing machine* from specifications, and use it to show that the TM accepts the language $a^n b^n c^n$.

Fuller specifications for the coding problems can be given on request.

References

- García, P., López, D., and Vázquez a de Parga, M. 2011. DFA minimization: from Brzozowski to Hopcroft. *Unknown*, p. Unknown.
- Hopcroft, J. E., Motwani, R., and Ullman, J. 2001. *Introduction to Automata Theory, Languages and Computation*. Addison-Wesley, 2nd edn.
- Valmari, A. 2010. $O(m \log n)$ time algorithms for DFA minimization and more. In *Lecture Slides*.