CSC 311—Theory of Computation

Tutorial on DFAs, NFAs, regular expressions, regular grammar, closure of regular languages, context-free grammars, non-deterministic push-down automata, Turing machines,etc.

Tutorial 2 6 September 2016

Assume that $\Sigma = \{a, b\}$ when it is not otherwise given.

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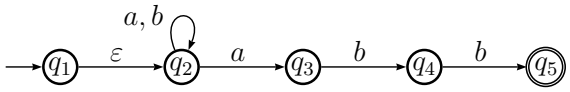
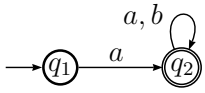
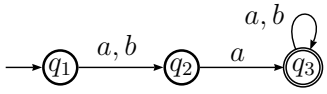
1. Given the regular expression `(a|b)*abb`, the corresponding NFA is given on the right. Rewrite this NFA into its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



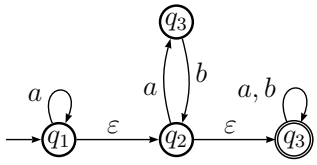
2. Given the NFA N with $L(N) = \{w \in \Sigma^* | w \text{ starts with } a\}$, as the regular expression `a(a+b)*`, first confirm that its NFA is as shown on the right then derive its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



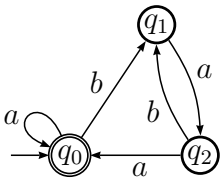
3. $L(N) = \{w \in \Sigma^* | w \text{ has } a \text{ as its 2nd letter}\}$ is the language of the NFA N. Write its corresponding regular expression. First confirm that its NFA is as shown on the right then derive its corresponding DFA. Show the resulting DFA both as a transition table and as a transition diagram.



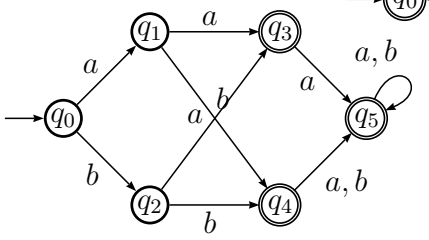
4. Consider NFA N diagrammed on the right. Describe $L(N) = \{w \in \Sigma^* | w \ldots\}$. Derive its corresponding DFA and write the resulting DFA both as a transition table and as a transition diagram.



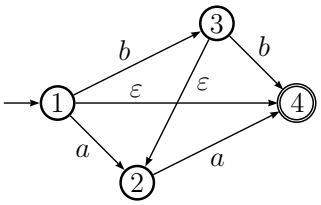
5. Give a regular expression for this NFA given as a transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



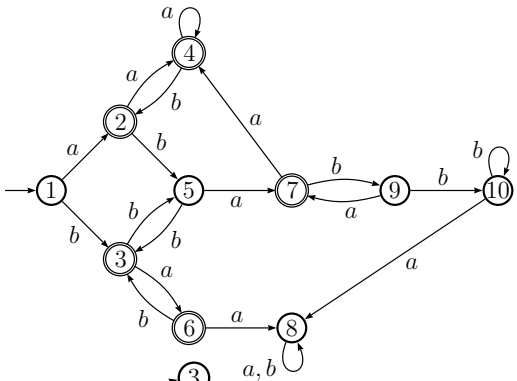
6. Minimize the DFA given by this transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



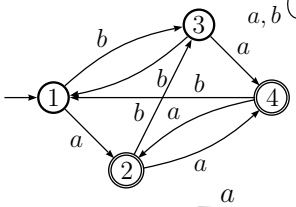
7. Rewrite the NFA given by the transition graph as a DFA, and then minimize the DFA. Show the resulting DFA both as a transition table and as a transition diagram.



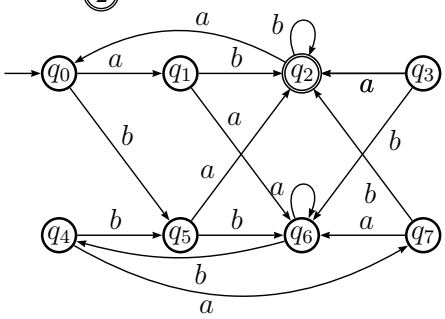
8. Minimize the DFA given by the transition graph (García et al., 2011). Show the resulting DFA both as a transition table and as a transition diagram.



9. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



10. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 155).



11. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 164).

	a	b
$\rightarrow 0$	1	0
1	0	3
2	3	1
*3	3	0
4	3	5
5	6	4
6	5	6
7	6	3

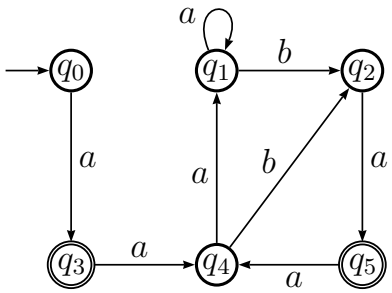
12. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 165).

	a	b
$\rightarrow 1$	2	5
2	3	6
*3	4	8
4	5	8
5	6	9
6	7	2
7	8	2
8	9	3
*9	1	5

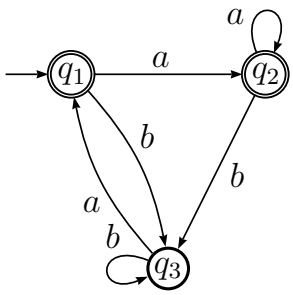
13. Minimize the DFA given by the transition table. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 165).

	a	b
$\rightarrow 0$	1	4
1	2	5
2	3	7
*3	4	7
4	5	8
5	6	1
6	7	1
7	8	2
8	0	4

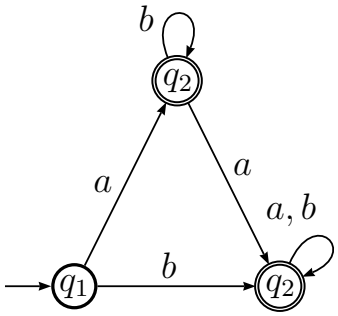
14. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Valmari, 2010). Note that all the missing transitions lead to a "dead" or "sink" state, which is not shown in the state diagram.



15. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram (Hopcroft et al., 2001, Page 158).



16. Minimize the DFA given by the transition graph. Show the resulting DFA both as a transition table and as a transition diagram.



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17. Show that the sentence `Jim trains art students` has a derivation in the following grammar. Give a parse tree for the sentence.

$$
\begin{aligned}
S &\rightarrow NV_P \\
N &\rightarrow NN \\
V_P &\rightarrow VN \\
N &\rightarrow \texttt{students} \mid \texttt{Jim} \mid \texttt{art} \mid \texttt{trains} \\
V &\rightarrow \texttt{trains}
\end{aligned}
$$

18. Let G be the following grammar

$$
\begin{aligned}
S &\rightarrow AB \mid BC \\
A &\rightarrow BA \mid \texttt{a} \\
B &\rightarrow CC \mid \texttt{b} \\
C &\rightarrow AB \mid \texttt{a}
\end{aligned}
$$

Use the CYK algorithm to determine whether $w = \texttt{baaba}$ is in L(G). Give a parse tree for w, and its rightmost derivation from S

19. Given the grammar G with start symbol E in CNF use the CYK algorithm to determine whether $w = \texttt{(x + x) * x}$ is in G. E is the start variable.

$$
\begin{aligned}
E &\rightarrow EF \mid LR \mid \texttt{x} \\
F &\rightarrow ME \mid CE \\
G &\rightarrow EQ \\
P &\rightarrow \texttt{+} \\
T &\rightarrow \texttt{*} \\
L &\rightarrow \texttt{(} \\
R &\rightarrow \texttt{)}
\end{aligned}
$$

20. Given the grammar G with start symbol S in CNF use the CYK algorithm to determine whether $w =$ `ababa` is in $L(G)$.

$$\begin{aligned} S &\rightarrow AT \mid AU \mid \varepsilon \\ T &\rightarrow UB \mid \texttt{b} \\ U &\rightarrow AT \mid UT \\ A &\rightarrow \texttt{a} \\ B &\rightarrow \texttt{b} \end{aligned}$$

21. Show the CYK algorithm for the CNF grammar G with start symbol S by showing that $w =$ `ababa` is or is not in $L(G)$.

$$\begin{aligned} S &\rightarrow AB|BC \\ A &\rightarrow BA|\texttt{a} \\ B &\rightarrow CC|\texttt{b} \\ C &\rightarrow AB|\texttt{a} \end{aligned}$$

22. What is the language $\mathscr{L}$ that is derived from S where

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aA \mid \texttt{a} \\ B &\rightarrow AB? \end{aligned}$$

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23. Write python code to read in the specifications for an NFA or DFA and then turns it into a DFA if it is not already one. Your specifications should give the NFA/DFA in the form of a *state table* with numerically numbered states and single characters as the alphabet. Use a python's sets, dictionary and regular expression package `re`.

24. Write python code which which *minimizes* your DFA.

25. Write python code to *turn a context-free grammar into Chomsky-normal form* if it is not yet in that form.

26. Write python code to *implement the CYK algorithm*, given a grammar in Chomsky normal form. It should read a grammar and run CYK to determine if some strings are accepted or rejected.

27. Develop code to *implement a PDA* from specifications, and use it to show that the PDA accepts the palindromes language.

28. Develop code to *implement a Turing machine* from specifications, and use it to show that the TM accepts the language $a^n b^n c^n$.

Fuller specifications for the coding problems can be given on request.

References

García, P., López, D., and Vázquez a de Parga, M. 2011. DFA minimization: from Brzozowski to Hopcroft. *Unknown*, p. Unknown.

Hopcroft, J. E., Motwani, R., and Ullman, J. 2001. *Introduction to Automata Theory, Languages and Computation*. Addison-Wesley, 2nd edn.

Valmari, A. 2010. $O(m \log n)$ time algoritms for DFA minimization and more. In *Lecture Slides*.