University of the Western Cape

Department of Computer Science

Theory of Computation—COS311

Final Examination 26 October 2015

Time: 3 hours. Marks: 120.

UWC number	Surname, first name

No calculators or cellphones are allowed. Please answer all the questions directly on this question paper. No other answers will be marked.

1. (a), *First*, for (1) mark, *give* one example of the numbers described below that is negative, has at least ten characters and a positive exponent and then (b) give a *regular expression* in the style of `flex` for:

   a *decimal floating point number* that starts with an *optional* '`-`', and has at most one digit before a *required* decimal point, '`.`', and from one to eight digits after the point. This is followed by an *optional exponent*: that starts with a *compulsory* exponent indicator in the form of an '`e`', or '`E`', followed by either a plus or a minus sign before one, two or three digits. A correct `flex` regular expression earns (9) marks. [10]

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2. Answer `true` or `false`: One (1) mark is *deducted* for each incorrect answer, so DO NOT GUESS. [15]

   (a) $ww^{\mathcal{R}}$ is a context-free language, where $w \in \Sigma^*$ and $\Sigma = \{a, b\}$. (1) ....

   (b) ww is a context-free language, where $w \in \Sigma^*$ and $\Sigma = \{a, b\}$. (1) ....

   (c) a^*b^* is a context-free language. (1) ....

   (d) a^nb^n is a regular language. (1) ....

   (e) $a^nb^n \subseteq a^*b^*$. (1) ....

   (f) The set of regular languages $\subseteq$ the set of context-free languages. (1) ....

   (g) If L_1 and L_2 are context free, then $L_1 \circ L_2$ is context free. (1) ....

   (h) If L_1 and L_2 are context free, then $L_1 \cap L_2$ is context free. (1) ....

(i) If L is context free, then L is Turing recognizable. (1) ....

(j) The language $\{a^j b^k c^\ell \mid j \geq 0, k \geq 0, \ell \geq 0\}$ is regular. (1) ....

(k) The language $\{a^j b^k c^\ell \mid j \geq 0, k \geq 0, \ell \geq 0, j = \ell\}$ is context free. (1) ....

(l) The set $A_{REX} = \{\langle R, w\rangle \mid R$ is a regular expression that generates the input string $w\}$ is decidable. (1) ....

(m) The set $A_{CFG} = \{\langle G, w\rangle \mid G$ is a CFG that accepts the input string $w\}$ is decidable. (1) ....

(n) The set $A_{TM} = \{\langle M, w\rangle \mid M$ is a TM that accepts the input string $w\}$ is decidable. (1) ....

(o) The set $\overline{A_{TM}}$, the complement of A_{TM}, is Turing recognizable. (1) ....

3. In these questions you must fill in a short answer as a number or a string of symbols or letters. [5]

(a) Suppose that the rational numbers are enumerated using the mapping for J below to map $\frac{p}{q}$ onto the natural numbers. Give the values of p and q that map onto 49.

$J[p, q] = (p + q - 1)(p + q - 2)/2 + q$ (1) p = ... (1) q = ...

(b) Give a regular expression for $\{a^2 b^2 c^n | n \geq 1\}$ or write the word *impossible.* (1) ....

(c) Given the length of a parsable string $|w| = n$, what is the height of the parse tree found for it when applying the CYK algorithm. (1) ....

(d) Given the rules $\mathscr{R} \equiv \{S' \to aSbSc, S \to \varepsilon\}$, after removing the $S \to \varepsilon$, how many rules must be added to make the grammar ε-free? (1) ....

(e) Given the same set of rules $\mathscr{R} \equiv \{S' \to aSbSc, S \to \varepsilon\}$, add rules to make the grammar ε-free and then alter the rules to turn the grammar into Chomsky normal form. [5]

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4. (a) Given a regular language L, the pumping lemma states that all strings $s \in L$ can be pumped if $|s| \geq p$, where p is the pumping length. The result of the pumping lies in L. The pumping repeats a section of the string $s = xyz$ under the following conditions:

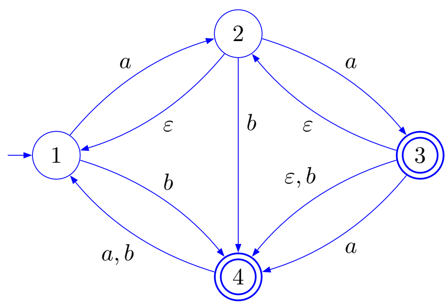
1. for any $i \geq 0$, $xy^iz \in L$,
2. $|y| \geq 0$, and
3. $|xy| < p$.

If the pumping lemma for regular languages, above, *is stated wrongly* please correct all the errors by marking them clearly *above directly on the question paper*, before proceeding with the rest of the question.(2) Let $\Sigma = \{a, b\}$. Is the language $L = \{a^nb^n \mid n \geq 0\}$ regular/not regular? Prove it using the corrected pumping lemma.(8) [10]

5. Give a regular expression for the language over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$ that accepts all strings that end with exactly two as. [2]

6. Convert the regular expression $\texttt{(a+b)}^2\texttt{b}^*\texttt{a}^*\texttt{(a+b)}^2\texttt{b}^*$ to an NFA in the form of a state diagram. [3]

7. Consider the state diagram below



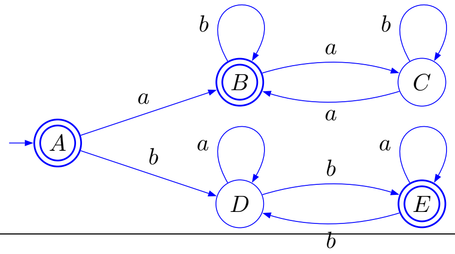
(a) What is the ε-closure of each of the states of the above NFA? [2]

(b) Use the subset construction to build a *transition table* for a DFA accepting the same language as the above NFA in the state diagram above. [8]

8. (a) Describe the production rules of *context free grammars*. [2]

(b) Describe the production rules of *regular grammars*. [2]

(c) Let L be the language recognized by the DFA to the right. Give the productions of a *regular* CFG used to generate the language $L - \{\varepsilon\}$. [3]



(d) Give a regular expression for the ε-free language $L - \varepsilon$, recognized by the above state diagram in Question 8(c). [3]

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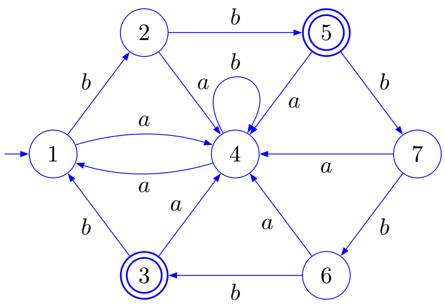
9. Find the *minimal* DFA equivalent to the following DFA described by the state diagram:



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.	.	3				
.	.	.	4			
.	.	.	.	5		
.	.	.	.	.	6	
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First, mark in the table on the right all the states that are not equivalent.

Now give the minimal DFA as a transition TABLE. [10]

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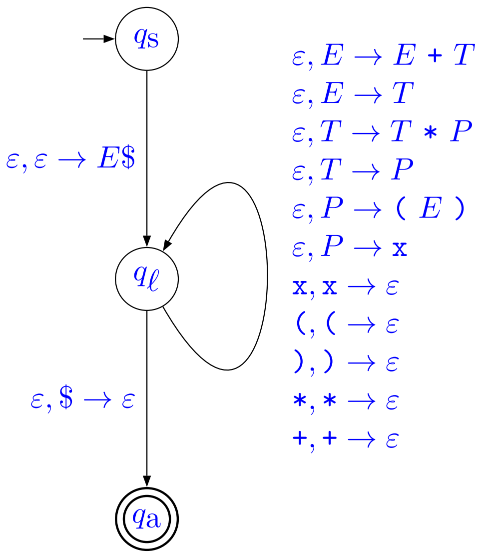
Use this box for your rough work.

10. Given the state diagram below for a nondeterministic PDA that recognizes a language, give a context free grammar G for this language. [3]



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11. The language $D = \{x^{\mathcal{R}}x | x \in \{a, b\}^*\}$ is context free. Illustrate two different proofs of this fact. [5]

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12. Use the Cocke-Younger-Kasami algorithm to decide if the string $v =$ `vuwuxyu` is generated by the grammar G, i.e., $v \in L(G)$, and that the string $w =$ `uwxuyuv` is *not* generated by the grammar. The grammar G is given by the following rules.

$$
\begin{aligned}
U &\rightarrow US \mid VT \mid \texttt{u} \\
S &\rightarrow WU \mid YU \\
T &\rightarrow UX \\
W &\rightarrow \texttt{w} \\
Y &\rightarrow \texttt{y} \\
V &\rightarrow \texttt{v} \\
X &\rightarrow \texttt{x}
\end{aligned}
$$

where U is the start symbol. First *complete* the following tables: (8)

Use this table to show that $v \in L(G)$.

0							
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Use this table to show that $w \notin L(G)$.

0							
.	1						
.	.	2					
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Now write out a detailed *rightmost* derivation of $U \overset{*}{\Rightarrow} v$. (2) [10]

13. Consider the following Turing machine, then answer the questions using this machine

δ	0	1	x	#	$\sqcup$
$\rightarrow q_1$	$q_2, \mathtt{x}, R$	$q_3, \mathtt{x}, R$	$q_R, \mathtt{x}, R$	$q_8, \#, R$	$q_R, \sqcup, R$
q_2	$q_2, 0, R$	$q_2, 1, R$	$q_R, 1, R$	$q_4, \#, R$	$q_R, \#, R$
q_3	$q_3, 0, R$	$q_3, 1, R$	$q_R, 1, R$	$q_5, \#, R$	$q_R, \#, R$
q_4	$q_6, \mathtt{x}, L$	$q_R, 1, R$	$q_4, \mathtt{x}, R$	$q_R, \#, R$	$q_R, \sqcup, R$
q_5	$q_R, 0, R$	$q_6, \mathtt{x}, L$	$q_5, \mathtt{x}, R$	$q_R, \#, R$	$q_R, \sqcup, R$
q_6	$q_6, 0, L$	$q_6, 1, L$	$q_6, \mathtt{x}, L$	$q_7, \#, L$	$q_R, \sqcup, R$
q_7	$q_7, 0, L$	$q_7, 1, L$	$q_1, \mathtt{x}, R$	$q_R, \mathtt{x}, R$	$q_R, \sqcup, R$
q_8	$q_R, 0, R$	$q_R, 1, R$	$q_8, \mathtt{x}, R$	$q_R, \mathtt{x}, R$	$q_A, \sqcup, R*$

(a) Show the moves this Turing machine makes while processing the string $10\#10\sqcup\ldots$ after starting at the start state. Show all the configurations the machine traverses through to accept the string. [6]

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(b) In order to process the string $0\#01\sqcup\ldots$, starting in the configuration $q_1 0\#01$, show the configurations the machine goes through to *reject* this string. [3]

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14. What is meant by *Turing-recognizability*. [3]

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15. What is a *decider*? Give an example of a decider. [3]

16. Every context-free language is decidable. Sketch, i.e., describe briefly, a proof for this. [3]

17. Consider the Turing recognizability of

$$D = \{P | P \text{ is a polynomial over } x \text{ with an integral root}\}?$$

Is D Turing recognizable? Answer YES or NO (1), and give an explanation (3). [4]

Total [120]